20. PLOTNIKOV V.A., Averaging differential inclusions. Ukr. Mat. Zh., 31, 5, 1979.
21. HARTMAN F., Ordinary differential equations. Moscow, Mir, 1970.
22. GUSEINOV G.KH., SUBBOTIN A.I. and USHAKOV V.N., Derivatives of multiple-valued mappings and their application in game problems of control. Probl. Control Inform. Theory, 14, 3, 1985.
23. PANASYUK A.I., Quasidifferential equations in metric space. Differents. Uravneniya, 21, 8, 1985.
24. PANASYUK A.I. and PANASYUK V.I., Asymptotic main-line optimization of controllable systems. Minsk: Nauka i Tekhnika, 1986.

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OPTIMAL ESTIMATES OF THE COORDINATES OF SYSTEMS WITH A TIME LAG WITH RESPECT TO A SET OF CONTINUOUS AND DISCRETE OBSERVATIONS*

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Expressions for optimal estimates of coordinates of systems with a time lag when there are continuous and discrete measurements are established and investigated. The effect of the amount of lag on the quality of estimation is demonstrated as an example. The related problems of when there are only continuous measurements were considered previously /1, 2/.

1. Formulation of the problem. Consider a dynamic system whose motion in the segment $[0, T]$ is described by a stochastic equation with initial conditions

$$x^{\cdot}(t) = A(t)\,x(t-h_1) + \sigma_1(t)\,\xi_1^{\cdot}(t),\ 0 \leqslant t \leqslant T \tag{1.1}$$

$$x(s) = 0,\ s < 0;\ x(0) = x_0 \tag{1.2}$$

For system (1.1) we carry out the following continuous $y(t)$ and discrete y_i observations at specified instants of time t_i

$$y(t) = g(t)\,x(t-h) + \sigma_2(t)\,\xi_2^{\cdot}(t),\ \ 0 \leqslant t \leqslant T \tag{1.3}$$

$$y_i = \beta_i x(t_i) + r_i\zeta_i,\ 0 \leqslant t_i \leqslant T,\ i=1,\ldots,N;\ t_1 < t_2 < \ldots < t_N \tag{1.4}$$

In Eqs.(1.1)-(1.4) the phase vector $x \in R_n$ (where R_n is an n-dimensional Euclidean space); the matrices A, σ_1, g, σ_2 with piecewise-continuous elements and the matrices β_i and r_i are specified; the time-lag constants $h_1, h \geqslant 0$; the Gaussian random quantities with zero expectation and unit covariation matrix are denoted by ζ_i, and ξ_1 and ξ_2 are standard Wiener processes; the Gaussian random quantity x_0 is such that $Mx_0 = 0$, $D_0 = Mx_0x_0'$. Here M is the sign of expectation, the prime is the sign of transposition, and D_0 is a specified positive-definite matrix. The random quantities $\xi_1, \xi_2, x_0, \zeta_i$ are mutually independent. Finally, without loss of generality, it is assumed that $y \in R_n$, $y_i \in R_n$.

Note that consideration of the time lag in the channel of measurements (1.3) is caused by the finiteness of time necessary to carry out the observations and to work out their results.

The need to consider the time lag in a measurement channel has been noted repeatedly in applied work (e.g. /3/). The choice of the initial conditions in the form (1.2) indicates that the motion of the system is only described by Eqs.(1.1) for $t \geqslant 0$, and nothing is known regarding the system when $t < 0$. In accordance with this, the continuous observations (1.3) that can be produced in the segment of time $[0, h]$ cannot carry any data about the system either, which was reflected in the above assumptions.

The problem consists of constructing an estimate $m(T)$ - which is optimal in the mean-square sense - of the vector $x(T)$, using the results of the observations (1.3), (1.4) in the segment $[0, T]$. It is known that $m(T)$ is the conditional expectation $x(T)$ under the conditions

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$y(t)$, $0 \leqslant t \leqslant T$, $y(t_i)$, $i = 1, \ldots, N$. The corresponding matrix of covariation of the conditional distribution

$$D(T) = M[x(T) - m(T)][x(T) - m(T)]'$$

Note that the problem of estimating the coordinates $x(t)$ of a system whose motion is described by an equation of the form

$$x^{\cdot}(t) = A_1(t)x(t) + A(t)x(t-h_1) + \sigma_1(t)\xi_1^{\cdot}(t)$$

which is more common compared with (1.1), also reduces to the problem formulated above.

For this it is sufficient to introduce new coordinates using the non-degenerate transformation $x_1(t) = Z_1(0, t)x(t)$, where $Z_1(t, s)$ is the fundamental matrix of the system $x^{\cdot}(t) = A_1(t)x(t)$. At the same time $x_1(t)$ is described by an equation of the form (1.1).

Below it is assumed that the matrices $\sigma_2(t)\sigma_2'(t)$ and r_ir_i' are non-degenerate for all $t \in [0, T]$ and $i = 1, \ldots, N$. In cases when the arguments of the above functions agree with t, they are sometimes omitted.

The above problem when $h = 0$ (i.e. when there is no time lag in the channel of observations) has been the subject of a number of analyses (e.g., /4/, which has an extensive bibliography). The case of arbitrary $h \geqslant 0$ was analysed only for continuous observations.

2. Filtration. The method of constructing the functions $m(T)$ and $D(T)$ is based on reducing the estimation problem to one of the optimal control of a linear determinate system with a quadratic minimisable functional. In view of the Gaussian nature of the conditional distribution $x(T)$ using the result of the observations (1.3), (1.4) the optimal estimate $m(T)$ is represented in the form /5/

$$m(T) = \int_0^T u(t)y(t)\,dt + \sum_{i=1}^N v_iy_i \tag{2.1}$$

The square matrices $u(t)$ and v_i are to be determined from the minimum condition of the error of estimation. We shall use $Z(t, s)$ to denote the fundamental matrix of system (1.1), which is determined by the relations (I is the unit matrix)

$$\partial Z(t, s)/\partial t = A(t)Z(t-h_1, s),\ t > s,\ Z(t, t) = I$$
$$\partial Z(t, s)/\partial s = -Z(t, s+h_1)A(s+h_1),\ Z(t, \tau) = 0,\ t < \tau$$

Then /6/

$$x(t) = \int_0^T Z(t, s)\sigma_1(s)\,d\xi_1(s) + Z(t, 0)x_0 \tag{2.2}$$

The stochastic integrals encountered are understood in Ito's sense. It is convenient to assume

$$u(t) = 0,\ g(t) = 0,\ A(t) = 0,\ t \overline{\in} [0, T] \tag{2.3}$$

Instead of $y(t)$ and y_i we shall substitute into (2.1) their expressions (1.3) and (1.4), and shall replace $x(t)$ in the resulting relation in accordance with (2.2). Bearing in mind (2.3), we finally have

$$\begin{aligned}x(T) - m(T) = \Big[&Z(T, 0) - \int_0^T u(t+h)g(t+h)Z(t, 0)\,dt - \\ &\sum_{i=1}^N v_i\beta_iZ(t_i, 0)\Big]x_0 + \int_0^T \Big[Z(T, s) - \\ &\int_s^T u(t+h)g(t+h)Z(t, s)\,dt\Big]\sigma_1(s)\,d\xi_1(s) - \\ &\sum_{i=1}^n v_i\beta_i\int_0^{t_i} Z(t_i, s)\sigma_1(s)\,d\xi_1(s) - \int_0^T u(t)\sigma_2(t)\,d\xi_2(t) - \sum_{i=1}^N v_ir_i\zeta_i\end{aligned}$$

Using this relation, we will obtain

$$\begin{aligned}M|x(T) - m(T)|^2 = \operatorname{Tr} D(T) = \operatorname{Tr}\Big[&\alpha'(0)D_0\alpha(0) + \\ &\int_0^T (\alpha'(t)\sigma_1(t)\sigma_1'(t)\alpha(t) + u'(t)\sigma_2(t)\sigma_2'(t)u(t))\,dt + \sum_{i=1}^N v_i'r_ir_i'v_i\Big] = J\end{aligned} \tag{2.4}$$

The determinate matrix $\alpha(t)$ is defined as the solution of the problem

$$\alpha^{\cdot}(t) = -A'(t+h_1)\alpha(t+h_1) + g'(t+h)u(t+h) + \tag{2.5}$$

$$\sum_{i=1}^{N} \beta_i' v_i \delta(t-t_i)$$

$$\alpha(\tau)=0, \quad \tau > T, \quad \alpha(T)=I$$

Thus, the matrices $u(t)$ and v_i in (2.1) must be determined by minimizing the functional (2.4) on the trajectories of system (2.5) under the conditions (2.3).

The control $u(t)$, $0 \leqslant t \leqslant h$ does not affect the behaviour of $\alpha(t)$; therefore bearing (2.4) in mind we have

$$u(t) = 0, \ 0 \leqslant t < h \tag{2.6}$$

The discrete optimal control v_i, and also the continuous control $u(t)$, $t \geqslant h$, based on the dynamic programming method, are constructed in the same way as in /2/. At the same time it is found that

$$D(t) = P(t) \tag{2.7}$$

$$v_i = (r_i r_i')^{-1} \beta_i \left[P(t_i+0)\alpha(t_i+0) + \int_{-h_1}^{0} Q(t_i+0, \tau)\alpha(t_i-\tau)\,d\tau \right] \tag{2.8}$$

The control $u(t)$ has the form

$$u(t) = (\sigma_2(t)\sigma_2'(t))^{-1} g'(t) \left[P(t-h)\alpha(t-h) + \int_{-h_1}^{0} Q(t-h, \tau)\alpha(t-\tau-h)\,d\tau \right] \tag{2.9}$$

$$T \geqslant t \geqslant h, \ u(t) = 0, \ 0 \leqslant t \leqslant h, \ t > T$$

The matrices $P(t), Q(t, \tau, R(t, \tau, \rho))$ for all $0 \leqslant t \leqslant T, t \neq t_i, -h_1 \leqslant \tau, \rho \leqslant 0$ satisfy the equations

$$B(t-h) = g'(t)(\sigma_2(t)\sigma_2'(t))^{-1} g(t) \tag{2.10}$$
$$P^{\cdot}(t) = Q(t, 0) + Q'(t, 0) + \sigma_1(t)\sigma_1'(t) - P(t)B(t)P(t)$$
$$R(t, 0, \tau) - (\partial/\partial t + \partial/\partial\tau) Q(t, \tau) - P(t)B(t)Q(t, \tau) = 0$$
$$(\partial/\partial t + \partial/\partial\tau + \partial/\partial\rho) R(t, \tau, \rho) + Q'(t, \tau)B(t)Q(t, \rho) = 0$$

At the jump points (i.e. when $t = t_i$) the following relations hold:

$$P(t_i+0) = P(t_i-0) - P(t_i-0)\gamma_i P(t_i+0) \tag{2.11}$$
$$Q(t_i+0, \tau) = Q(t_i-0, \tau) - P(t_i-0)\gamma_i Q(t_i+0, \tau)$$
$$R(t_i+0, \tau, \rho) = R(t_i-0, \tau, \rho) - Q'(t_i-0, \tau)\gamma_i Q(t_i+0, \tau)$$
$$\gamma_i = \beta_i'(r_i r_i')^{-1}\beta_i$$

The boundary conditions for Eqs.(2.10), (2.11) have the form

$$P(0) = D_0, \ Q(0, \tau) = R(0, \tau, \rho) = 0, \ -h_1 < \tau, \rho \leqslant 0 \tag{2.12}$$
$$A(t+h_1)P(t) - Q'(t_1, -h_1) = 0, \ 0 \leqslant t \leqslant T$$
$$2A(t+h_1)Q(t, \tau) - R(t, -h_1, \tau) - R'(t, \tau, -h_1) = 0$$

Thus, to construct the estimate $m(T)$ and the covariation matrix $D(T)$ we need:

1) to solve problem (2.10)-(2.12), having defined P and Q;

2) to solve problem (2.5) with the control v_i from (2.8) and the control $u(t)$ from Eq.(2.9), into which the calculated matrices P and Q are substituted;

3) to substitute P, Q, and α obtained into (2.8), (2.9), having thereby defined $u(t)$ and v_i;

4) $D(T)$ is now given by Eq.(2.7), and the estimate $m(T)$ is given by Eq.(2.1) for the values of v_i and $u(t)$ obtained.

We shall now consider cases in which the above filtration algorithm permits some simplifications.

3. The case $\sigma_1 = 0$. Then the equations of motion (1.1) are determinate and we can write an explicit representation for the matrices P, Q and R. We shall define the matrices $b(t)$ and $P_1(t)$ by means of the equations

$$b^{\cdot}(t) = b(t-h_1)A'(t), \ 0 \leqslant t \leqslant T, \ b(0) = I, \ b(\tau) = 0, \tag{3.1}$$
$$\tau < 0$$

$$P_1(t) = \left[\int_0^t b(s)B(s)b'(s)\,ds + \sum_{i: t_i < t} b(t_i)\gamma_i b'(t_i) + D_0^{-1} \right]^{-1}, \quad t \neq t_i \tag{3.2}$$

$$P_1(t_i+0) = [P_1^{-1}(t_i-0) + b'(t_i)\gamma_i b(t_i)]^{-1}$$

Then

$$P(t) = b'(t)P_1(t)b(t), \ Q(t, \tau) = b'(t)P_1(t)b^{\cdot}(t-\tau) \tag{3.3}$$

$$R(t, \tau, \rho) = b^{\cdot\prime}(t-\tau) P_1(t) b^{\cdot}(t-\rho), \quad -h \leqslant \tau, \rho \leqslant 0$$

At the point h_1 of discontinuity of the derivative $b^{\cdot}(t)$ the latter is determined by means of left-continuity. The matrix $b(t)$ is easily calculated in explicit form by integrating Eq.(3.1) with respect to steps of length h_1. The function $P_1(t)$ is considered to be left-continuous.

The validity of the representation (3.3) is verified by directly substituting (3.3) into (2.10)-(2.12). This verification is made easier by the fact that on the grounds of (3.2)

$$P_1^{\cdot}(t) = -P_1(t) b(t) B(t) b'(t), \quad t \neq t_i$$

4. The case when $h_1 = 0$. Then Eqs.(1.1) will not contain a time lag. The expressions for m and D depend on whether the motion of the system is only described by Eqs.(1.1) when $t \geqslant 0$, or these equations hold for all $t \geqslant -h$. We can obtain expressions for m and D when $h_1 = 0$ from the general relations of para.2. However the absence of a time lag in Eqs.(1.1) enables us to simplify the derivation of formulae for m and D. We shall first present them on the assumption that the motion of the system is described by Eqs.(1.1) for all $t \geqslant -h$, $h_1 = 0$ and only continuous observations are carried out. The initial condition has the form $x(0) = x_0$. We will assume $\varphi(t) = x(t-h)$. By virtue of Ito's formula and (1.1) we have

$$\varphi(0) = Z(-h, 0) x_0 + \int_{-h}^{0} Z(-h, t) \sigma_1(t) d\xi_1(t) \tag{4.1}$$

$$\varphi^{\cdot}(t) = A(t-h)\varphi(t) + \sigma_1(t-h)\xi_1^{\cdot}(t-h) \tag{4.2}$$

Suppose $m_1(t)$ and $D_1(t)$ are the conditional expectation and the covariation matrix of the vector $\varphi(t)$, provided that the vector $y(t) = g(t)\varphi(t) + \sigma_2(t)\xi_2^{\cdot}(t)$ is observed in the segment $[0, t]$. Then /4/

$$m_1^{\cdot}(t) = D_1 g(\sigma_2\sigma_2')^{-1}[y(t) - gm_1(t)] + A(t-h) m_1(t) \tag{4.3}$$

$$D_1^{\cdot}(t) = A(t-h) D_1 + D_1 A'(t-h) + [\sigma_1(t-h)\sigma_1'(t-h)]^{-1} - D_1 g'(\sigma_2\sigma_2')^{-1} g D_1 \tag{4.4}$$

The initial conditions for the set of Eqs.(4.3), (4.4) follow from (4.1). Namely

$$m_1(0) = Z(-h, 0) m_0 \tag{4.5}$$
$$D_1(0) = Z(-h, 0) D_0 Z'(-h, 0) + \int_{-h}^{0} Z(-h, t)\sigma_1(t)\sigma_1'(t) Z'(-h, t)\, dt$$

Using Ito's formula, we obtain

$$x(t) = Z(t, t-h) x(t-h) + \int_{t-h}^{t} Z(t, s)\sigma_1(s) d\xi_1(s)$$

Calculating the conditional expectation from both sides of this equation under the condition that $y(s)$, $0 \leqslant s \leqslant t$ is measurable, we obtain

$$m(t) = Z(t, t-h) m_1(t) \tag{4.6}$$

In a similar way, calculating the conditional covariation we have

$$D(t) = Z(t, t-h) D_1(t) Z'(t, t-h) + \int_{t-h}^{t} Z(t, s)\sigma_1(s)\sigma_1'(s) Z'(t, s)\, ds \tag{4.7}$$

Differentiating both sides of Eq.(4.6), (4.7) with respect to t, we obtain equations for $m(t)$ and $D(t)$ bearing in mind (4.3), (4.4). These equations are derived in exactly the same way in the general case of continuous and discrete observations (1.2), (1.3). It should merely be remembered that when there are discrete observations only the equation for $D^{-1}(t)$ remains valid. This equation - by which we mean an integral identity - has the form ($\delta(t)$ is the delta-function)

$$D(0) = D_0 \tag{4.8}$$
$$D^{\cdot -1}(t) + D^{-1}A + A'D^{-1} - \sum_{i=1}^{N} \beta_i'(r_i r_i')^{-1}\beta_i \delta(t - t_i) =$$
$$Z_1(t) - Z_1\bar{\sigma}_1 D^{-1} - D^{-1}\bar{\sigma}_1 Z_1 - D^{-1}[\sigma_1(t)\sigma_1'(t) - \bar{\sigma}_1 Z_1 \bar{\sigma}_1] D^{-1}(t)$$
$$Z_1(t) = Z'(t-h, t) g'(t)(\sigma_2(t)\sigma_2'(t))^{-1} g(t) Z(t-h, t)$$
$$\bar{\sigma}_1 = \int_{t-h}^{t} Z(t, s)\sigma_1(s)\sigma_1'(s) Z'(t, s)\, ds = \bar{\sigma}_1(t)$$

The initial condition $D(0) = D_0$ for Eq.(4.8) follows from (4.5), (4.7). The equation for the estimate has the form

$$m(t) - m_0 - \int_0^t A(s)\,m(s)\,ds - \sum_{i:t_i \leqslant t} D(t_i + 0)\,\beta_i'(r_i r_i')^{-1} \times \tag{4.9}$$
$$[y_i - \beta_i m(t_i - 0)] = \int_0^t \Big(D(s) - \int_{s-h}^{s} \sigma_1(s_1)\,ds_1\Big) Z'(s, s-h)\,g'(s) \times$$
$$(\sigma_2(s)\,\sigma_2'(s))^{-1}\,(y(s) - g(s)\,Z(s-h, s)\,m(s))\,ds$$

The last term on the left-hand side of (4.9) can be transformed in accordance with one of the following formulae (/7, 222/):

$$D(t_i + 0)\,\beta_i'(r_i r_i')^{-1} = [D(t_i - 0)^{-1} + \beta_i'(r_i r_i')^{-1}\beta_i]^{-1}\beta_i'(r_i r_i')^{-1} = \tag{4.10}$$
$$D(t_i - 0)\,\beta_i'[r_i r_i' + \beta_i D(t_i - 0)\,\beta_i']^{-1}$$

Suppose, finally, $h_1 = 0$, and the system begins its motion at the instant of time $t = 0$, but is only described by Eq.(1.1) when $t \geqslant 0$ with the initial condition (1.2). Then the equations of the optimal filter have the form (4.8), (4.9), in front of the right-hand side of which there is the common factor $\chi(t - h)$, where $\chi(t) = 0$ when $t \leqslant 0$ and $\chi(t) = 1$ when $t > 0$.

Note that Eqs.(2.11), (4.8) can be interpreted as a method of constructing solutions of the equations for $P(t)$ and $D(t)$, in which the jump points of the trajectory and control match.

5. Extrapolation. The extrapolation problem consists of constructing the best estimate, in the mean-square sense, of the next coordinates of system (1.1) at the instant $\tau > T$ using the results of the observations (1.3), (1.4) in the segment $[0, T]$. The solution of this problem reduces to a solution of the problem of filtration, obtained in Sect.2, in the following way. We shall introduce the function $g_0(t) = g(t)$ when $0 \leqslant t \leqslant T$ and $g_0(t) \equiv 0$ when $t > T$. Consider the subsidiary problem of the filtration of the vector $x(\tau)$ using the results of the measurements (1.3), (1.4) in the segment $[0, \tau]$, whilst we have $g_0(t)$ in (1.3) instead of $g(t)$. The vector $x(s)$, $0 \leqslant s \leqslant \tau$ is described by Eq.(1.1) with the initial conditions (1.2). In view of the independence of the random quantities x_0, ξ_1, ξ_2, ζ_i the solution of the problem of extrapolation is the same as that of the subsidiary problem of filtration, established in Sect.2, where one should replace T everywhere by $\tau\, m$, and $g(t)$ by $g_0(t)$.

6. Interpolation. The problem of interpolation consists of an optimal estimate of the preceding state of the vector $x(\tau_0)$, $0 \leqslant \tau_0 \leqslant T$ using the results of the observations (1.3), (1.4) in the segment $[0, T]$. The method of constructing the estimate $m(\tau_0)$ of the vector $x(\tau_0)$ remains the same as in para.2. At the same time it is found, like (2.1), that

$$m(\tau_0) = \int_0^T u(t)\,y(t)\,dt + \sum_{i=1}^N v_i y_i \tag{6.1}$$

The matrices $u(t)$ and v_i which figure in (6.1) are determined from the condition of the minimum of the functional (2.4) on the trajectories of the controllable system

$$\alpha^{\cdot}(t) = -A'(t + h_1)\,\alpha(t + h_1) + g'(t + h)\,u(t + h) + \tag{6.2}$$
$$\sum_{i=1}^N \beta_i v_i \delta(t - t_i)$$
$$0 \leqslant t \leqslant \tau_0,\ \tau_0 \leqslant t \leqslant T$$
$$\alpha(s) \equiv 0,\ s \geqslant T,\ \alpha(\tau_0 - 0) = I + \alpha(\tau_0 + 0) \tag{6.3}$$

System (6.2) does not greatly differ from system (2.5) by the equations of motion and the initial conditions, which is a consequence of the characteristics of the interpolation problem.

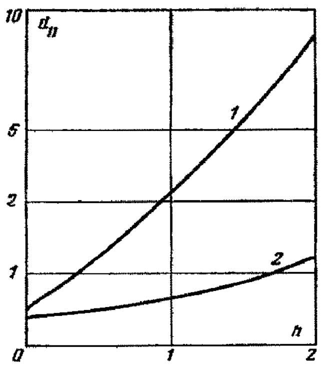
Remarks. 1°. The above shows that the characteristics of the specific estimation problem only appear in the form of equations of a dual controllable determinate system. We shall present them for the problem of estimating the vector $x(\tau_0)$, $0 \leqslant \tau_0 \leqslant T$ using the changes y_i and $y(t)$, $0 \leqslant t \leqslant T$, assuming that

$$x^{\cdot}(t) = \sum_{i=1}^N A_i(t)\,x(t - h_i) + \sigma_1(t)\,\xi_1^{\cdot}(t),\quad 0 \leqslant t \leqslant T,\quad h_i \geqslant 0$$
$$y(t) = \sum_{i=1}^N g_i(t)\,x(t - \tau_i) + \sigma_2(t)\,\xi_2^{\cdot}(t),\quad y(0) = 0,\quad \tau_i \geqslant 0$$

The discrete observations y_i satisfy Eqs.(1.4). The initial conditions for x have the form (1.2). The required estimate is determined using Eq.(2.1), in which the matrices $u(t)$ and v_i are determined from the condition for a minimum of the functional (2.4) on the trajectories of the system

$$\alpha^{\cdot}(t)=\sum_{i=1}^{N}[-A_i'(t+h_i)\,\alpha(t+h_i)+g_i'(t+\tau_i)\,u(t+\tau_i)+\beta_{i,*}'v_i\delta(t-\tau_i)]$$

The initial conditions for $\alpha(t)$ and the conditions of the jump when $t=\tau_0$ are specified by Eqs.(6.3).



2°. The dependence of the error of the estimation $D(T)$ on the amount of time lag h in the channel of measurements is interesting. It follows from the general relations of Sect.2 that an increase in h can lead both to an increase in $D(T)$, and to a reduction. As an example we shall present a graph of the dependence on h of the variance of the coordinate d_{11} of a free moving material point. The equations of motion (1.1) have the form

$$x_1^{\cdot}(t)=x_2(t),\ x_2^{\cdot}(t)=0,\quad -h\leqslant t\leqslant T,\ x(0)=x_0$$

The velocity is observed, i.e. $y(t)=x_2(t-h)+\sigma_2\xi^{\cdot}(t),\ 0\leqslant t\leqslant T$.

The expression for d_{11} is determined using Eq.(4.7). The dependence of d_{11} on h is represented in the figure, where curve *1* corresponds to the values of the parameters

$$T=2,\quad \sigma_2=1,\quad D_0^{-1}=\begin{Vmatrix}1&0\\0&1\end{Vmatrix}$$

and for curve *2*

$$T=2,\quad \sigma_2^{-2}=0{,}5,\quad D_0^{-1}=\begin{Vmatrix}2&0\\0&1\end{Vmatrix}$$

REFERENCES

1. KALMANOVSKII V.B., The filtration of some stochastic processes with a time lag. Avtomatika i Telemekhanika, 1, 1974.
2. KOLMANOVSKII V.B. and MAIZENBERG T.L., Optimal estimates of the state of a system and some problem of controlling equations with a time lag. PMM, 41, 3, 1977.
3. VAN TREES H., Theory of detection, estimation and modulation. 3, Moscow, Sov. Radio, 1977.
4. BRUMMER K. and ZIFFLING G., The Kalman-Bucy filter. Moscow, Nauka, 1982.
5. GIKHMAN I.I. and SKOROKHOD A.V., Introduction to the theory of random processes. Moscow, Nauka, 1977.
6. KOLMANOVSKII V.B. and NOSOV V.R., Stability and periodic modes of controllable systems with a time lag. Moscow, Nauka, 1981.
7. AOKI M., Optimization of stochastic systems. Moscow, Nauka, 1971.

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